Exploring Montessori Math Materials and Their Impact on Math Fluency: A Qualitative Action Research Study

By

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A Master's Paper Submitted in Partial Fulfillment of

The Requirements for the Degree of

Master of Science in Education – Montessori

May 5, 2024

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Abstract

In traditional Montessori classrooms, mastery of mathematical concepts is fostered using concrete, hands-on manipulatives (Snyder et al., 2021). At the same time, the increasing digitization of classrooms, particularly in the form of interactive software invites a potentially disruptive change (Scippo & Ardolino, 2021). Interactive software, designed to increase math fluency, offers an alternative to Montessori's tactile approach, creating a potential point of contention between traditional and modern pedagogical tools (Owen & Davies, 2020). Prior research has shown students in Montessori schools perform at or below those of traditional schools on standardized tests designed to measure math fluency (Basargekar and Lillard, 2021; Brown & Lewis, 2016; Hurdle, 2016; Snyder, 2021). Moreover, the math performance of students in the United States has significantly declined since 2018, with current scores being the lowest in two decades (*NAEP Mathematics: National Achievement-Level Results*, n.d.) .Meanwhile, the use of interactive computer software has been shown to be an effective way to increase math fluency of students in traditional classroom environments (Kurvinen et al., 2020). However, more research is needed to understand the use of Montessori math materials and their impact on math fluency in the elementary classroom and identify areas in which updates to the methodology and materials such as interactive software may be warranted. Therefore, using qualitative interviews, this study examines Montessori math materials and their impact on math fluency. Ultimately, the findings from this study will provide educators with optimized teaching strategies that meet the needs of 21st-century students. This study found: a) Montessori math materials are highly valued for facilitating the transition from concrete to abstract understanding through hands-on manipulatives, supporting tailored learning experiences, and promoting student autonomy, (b) challenges with Montessori materials include the need for repetition and difficulties in addressing specific mathematical concepts, with some educators advocating for the incorporation of

supplementary materials and approaches to overcome these limitations, and (c) the integration of non-Montessori materials and novel approaches, such as interactive software and other digital resources, is embraced by educators aiming to align with Montessori principles while enhancing engagement and real-world applicability of math concepts. Ultimately, this study illuminates the strengths and limitations of traditional Montessori math materials in fostering math fluency, highlighting the potential for incorporating modern educational technologies and methodologies to complement and enhance Montessori pedagogy. By offering insights into optimized teaching strategies that cater to the needs of 21st-century learners, the research contributes to the evolving discourse on balancing traditional Montessori methods with innovative educational tools to better serve elementary students' learning experiences.

Keywords: Montessori math materials, math fluency, elementary education, interactive software, educational innovation

Literature Review

This literature review explores outcomes and potential problems associated with the traditional Montessori approach to math fluency, along with the use of technology for increasing math fluency. This review contributes to advancing knowledge about Montessori education, particularly in the context of mathematical competency in Lower Elementary Montessori students. Moreover, this exploration of Montessori approaches to math fluency holds the potential for positive educational change for educators in Montessori schools. By providing evidence-based insights, educators can optimize teaching strategies and enhance student learning experiences, fostering an enriched learning environment in Montessori classrooms.

What is Math Fluency?

The National Council of Teachers of Mathematics (NCTM) highlights the importance of mathematics skills for students, noting their foundational role in education and professional success (*Policies and Recommendations - National Council of Teachers of Mathematics*, n.d.). According to the NCTM:

> Fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate *flexibility* in the computational methods they choose, understand, and can explain these methods, and produce *accurate* answers *efficiently*. The computational methods that a student uses should be based on mathematical ideas that the student understands well, including the structure of the base-ten number system, properties of multiplication and division, and number relationships. (The National Council of Teachers of Mathematics, Inc. et al., 2000, p. 152)

Research has focused on the concept of automaticity, defined by Stickney et al. (2012) as the ability to promptly recall correct answers from memory without conscious calculation. Automaticity is considered a key aspect of mathematical fluency (Stickney et al., 2012). The

progression from developing automaticity to achieving fluency is suggested to foster long-term success in students' mathematics education (Cumming & Elkins, 1999; Stickney et al., 2012; Woodward, 2006).

The Montessori Approach to Math Fluency

Math Fluency Outcomes of the Montessori Approach

The Montessori approach to education, which emphasizes hands-on and experiential learning, faces potential challenges in the realm of math fluency, as indicated by current research findings (Basargekar and Lillard, 2021; Brown & Lewis, 2016; Hurdle, 2016; Snyder, 2021). Hurdle (2016) studied the methods and educational experiences encountered by third and fourth-grade students who transitioned from Montessori to traditional schools and how the alteration of teaching methods and learning opportunities impacted the mathematical problem-solving proficiency of students. Hurdle (2016) demonstrated that students' challenges in procedural fluency, identified through assessment results, teacher discussions, and student feedback, could pose difficulties in their transition out of a Montessori mathematics program. Furthermore, the findings highlight that while these same students exhibit a robust conceptual understanding, their struggles with procedural fluency are exacerbated when they fail to actively construct their knowledge and understanding without the use of concrete materials.

Further examining Montessori outcomes, Snyder et al. (2021) conducted a study to determine whether the percentages of children meeting their state's proficiency standard at public Montessori schools differ from their traditional public-school counterparts. The study examined publicly available report card data from three schools and then analyzed the data through repeated measures analyses of covariance (ANCOVAs). The analysis included the consideration of covariates related to the percentage of students affected by the racial opportunity gap and

economically disadvantaged students (Snyder et al., 2021). Snyder et al. (2021) showed that Montessori schools may not be as effective as non-Montessori schools in improving standardized test proficiency for 3rd graders. Snyder et al. (2021) posit that Montessori children might depend on manipulatives to carry out math problems and therefore do poorly on standardized tests where manipulatives are not available. This reliance on manipulatives for computation might be particularly the case at younger ages as children in Montessori classrooms are said to use the materials less after around age 9 as they become more apt at abstraction (Snyder et al. 2021). It has also been suggested that because the manipulatives engender a deeper understanding, Montessori students excel on deeper conceptual math, problems but not on more superficial problems that appear on state standardized tests (Snyder et al. 2021). Snyder et al. (2021) point to the need for future studies to determine if younger Montessori children would perform as well as or better than traditionally schooled children on conceptual math questions and how the availability of the math materials during standardized tests would affect outcomes.

In another study examining outcomes of the traditional Montessori approach, Brown and Lewis (2016) studied the effectiveness of Montessori reading and math instruction for third-grade African American students in urban public schools. The study's treatment group comprised African American students who finished third grade in three public Montessori magnet schools in a large urban district in North Carolina. To create a comparison group, researchers selected third-grade African American students from families who had also self-selected into a choice program, attending similar magnet schools within the same attendance zones of the same district (Brown & Lewis, 2016). Brown and Lewis (2016) used a quantitative analysis using scores from North Carolina's End-of-Grade tests. Raw scores from three editions were converted to z-scores using the mean and standard deviation of each edition. The study indicated that attending Montessori schools may lead to notably better reading performance with potential benefits for African

American students, but math scores did not differ between types of schools. The lack of significant differences in math scores raises questions about the effectiveness of the Montessori method in teaching mathematics. Further research may be necessary to explore the specific aspects of Montessori math education and its potential impact on students.

Looking further into outcomes of the Montessori approach, Basargekar and Lillard (2021) reviewed research literature comparing the math achievement of children enrolled in Montessori or conventional schools. Basargekar and Lillard (2021) revealed three key findings in their research: (a) t Montessori advantage becomes more apparent when assessing deeper conceptual knowledge rather than procedural fluency, (b) t limitations of standardized measurements and end-of-grade tests may hinder a comprehensive evaluation of Montessori students' understanding, and (c) to gain a more comprehensive understanding of the mathematics learning process in Montessori schools, future investigations should explore the impact of the Primary (ages 3–6) Montessori experience on math learning in Elementary (ages 6-12) Montessori schools. Evaluating Montessori students' use of algorithms and strategies, possibly through application and transfer-oriented assessments, could provide valuable insights into their learning processes and conceptual understanding. The findings emphasize the need for future investigations to explore the impact of the Primary Montessori experience on math learning in Elementary Montessori schools, urging a shift towards application and transfer-oriented assessments for a more insightful examination of students' learning processes.

Potential Problems with the Montessori Approach to Math Fluency

As part of an exploration of the Montessori approach to math fluency, it is imperative to consider research findings that provide insights into the fundamental elements of the Montessori

approach to math fluency which includes the use of manipulatives, student-directed learning, and specialized materials. A study by Carbonneau (2013) offers a comprehensive analysis of the role of manipulatives in mathematics education, providing valuable insights into their impact on student learning. Carbonneau (2013) studied the efficiency of utilizing hands-on materials in mathematics education in comparison to teaching mathematics solely through abstract mathematical symbols. The analysis examined 55 studies that contrasted teaching with hands-on learning materials to a control group where mathematics was taught using only abstract mathematical symbols. The studies involved students ranging from kindergarten to college level (Carbonneau et al., 2013). Carbonneau et al. (2013) uncovered three primary findings from their study: (a) the utilization of manipulatives in mathematics instruction has a moderate impact on student learning when contrasted with instruction solely using abstract symbols, (b) the magnitude of this impact relies on various instructional factors, including the perceptual richness of objects, the level of guidance provided to students during the learning process, and the developmental stage of the learner, all of which influence the effectiveness of manipulatives, and (c) the revelation that specific instructional variables can either dampen or enhance the effectiveness of manipulatives suggests that merely incorporating manipulatives into mathematics instruction may not suffice to enhance students' achievement in mathematics, highlighting the importance of considering specific instructional variables to optimize the use of manipulatives in the classroom. In the context of Montessori education, which places a strong emphasis on hands-on, experiential learning and the use of manipulatives, this conclusion supports the idea that the success of the Montessori approach to math fluency is not solely reliant on the presence of manipulatives. Instead, it underscores the significance of how manipulatives are incorporated into the instructional process. Another fundamental component of the Montessori approach to math fluency is the emphasis on student-directed instruction over teacher-directed practice and drill. Underscoring the potential

drawback of this approach, Morgan et al. (2015) examined if teaching methods varied for first-grade students with and without a history of math difficulties, including temporary or ongoing issues. Morgan et al. (2015) found that only teacher-directed instruction rather than a student-directed approach was significantly linked to the achievement of students with math difficulties. Additionally, the most substantial predicted impact for a specific instructional approach was observed with routine practice and drill (Morgan et al., 2015). This study demonstrates that in first-grade classrooms with a higher percentage of students with mathematical difficulties, teachers should prioritize teacher-directed instruction over manipulative use to significantly improve the mathematical achievement of these students, while also utilizing routine practice and drill for the most substantial predicted impact. For non-math difficulty student groups, a balance between teacher-directed and student-centered instruction is recommended to achieve statistically significant positive predicted effects on overall student math fluency achievement (Morgan et al., 2015). In the context of the Montessori environment, the Montessori approach may align more closely with the needs of non-math difficulty student groups, where a balance between teacher-directed and student-centered instruction is deemed effective. However, for classrooms with a higher percentage of students with math difficulty, a more structured and teacher-directed approach, with an emphasis on routine practice and drill, is suggested to improve mathematical achievement.

The Montessori approach to math fluency places a great deal of emphasis on the use of didactic materials (Basargekar, A., & Lillard, A. S.,2021; Lillard, 2022). However, as Lillard (2011) revealed, there is a significant amount of variability in the use of these materials from one classroom to another. Lillard (2011) studied the viewpoints on a specific collection of Primary Montessori materials to pinpoint ones that provoke differing opinions among American Montessori teachers. This study demonstrates the challenges and discrepancies in the perceived necessity of

specific Montessori Math materials among different training programs. While there was a consensus on a core set of materials, significant variations were observed, particularly in the evaluation of Equation Booklets, which generated mixed feedback among AMI trainers, and unanimous disapproval of Math Workbooks and Worksheets across all trainers. The findings emphasize the need for a standardized approach to math materials in Montessori training programs. This suggests a recognition that a more consistent and uniform set of materials could enhance the effectiveness of Montessori educational practices, particularly in the realm of math fluency.

These studies collectively suggest potential pitfalls, including the need for careful consideration of instructional variables, a nuanced approach for students with mathematical difficulties, and the importance of standardizing Montessori math materials to ensure consistency and effectiveness in educational practices.

The Use of Technology for Increasing Math Fluency

The Effectiveness of Interactive Software and Math Fluency

The incorporation of interactive math software has been a topic of interest in educational research, particularly regarding its impact on the mathematical fluency of elementary students (Es-Sajjade&Paas, 2020; Fraga-Varela et al., 2021; Kacmaz and Dubé, 2022). Several studies have shed light on this theme, and their findings collectively suggest positive educational change for Montessori education. For example, Es-Sajjade and Paas (2020) found that students who interact with digital math games perform better than those learning through traditional methods. Moreover, even simple math games can lead to improved performance, and digital gaming does not affect motivation negatively (Es-Sajjade & Paas, 2020). These findings suggest that interactive software has the potential to engage students and enhance their math skills effectively (Es-Sajjade&Paas, 2020). Building on this idea, Fraga-Varela et al. (2021) explored the effects of serious games on

mathematics fluency in primary education students. A “serious game” is typically defined as a computer application that makes use of the mechanisms of video games to communicate specific information (knowledge) that helps introduce relevant concepts and the application of those concepts to solve problemsG (Fraga-Varela et al., 2021). Serious games differ from classical video games in that their primary objective is not entertainment but effective learning (Krath et al., 2021). The study by Fraga-Varela et al. (2021) revealed that serious games can improve students' understanding of math concepts, and classes that utilized gamification strategies showed increased progress in mathematics fluency. Furthermore, the study found a substantial correlation between performance test scores after serious gaming experiences and students' grades (Fraga-Varela et al, 2021). These findings indicate that incorporating interactive math software, especially serious games, can positively impact students' mathematical fluency and overall academic performance (Fraga-Varela et al, 2021). Complementing these findings, Kacmaz and Dubé (2022) conducted a systematic review of educational databases to understand the impact of math computer games on school-aged children. They discovered that games-based learning with direct instruction led to positive learning outcomes, highlighting the effectiveness of this approach (Kacmaz & Dubé,2022). However, learner-centered approaches like discovery, constructivism, and situated cognition were less represented in math games, suggesting a need for a more diverse range of pedagogical approaches in interactive math software (Kacmaz & Dubé, 2022). Additionally, Kurvinen et al. (2020) investigated the impacts of technology-enhanced learning (TEL) on the mathematical fluency of students. The study revealed that incorporating technology-driven lessons on a regular basis enhanced academic performance and served as a valuable supplement to conventional teaching methods (Kurvinen et al., 2020). The positive effects of effectively integrating technology into education were observed in both immediate and long-lasting outcomes (Kurvinen et al., 2020). This supports the idea that incorporating interactive math

software into the curriculum can lead to improved mathematical fluency and overall academic achievement (Kurvinen et al., 2020). Similarly, Gliksman et al. (2022) investigated mathematical fluency in primary school children and highlighted the significance of effective problem-solving, variations in math fluency across different operations, and the importance of consistent performance and assessment in supporting students' mathematical proficiency (Gliksman et al., 2022). This study reinforces the idea that interactive software can enhance problem-solving skills and improve mathematical fluency (Gliksman et al., 2022). Addressing specific challenges to math fluency, Baker and Cuevas (2018) found that students used strategies to solve single-digit multiplication problems but struggled to achieve automaticity. Baker and Cuevas (2018) conclude that regular use of such software can enhance math fact fluency and aid in developing automaticity, which is crucial for future success in mathematics.

In summary, studies consistently show that interactive software, especially serious games, can engage students, enhance problem-solving skills, and promote math fluency proficiency (Es-Sajjade & Paas, 2020; Fraga-Varela et al., 2021; Kacmaz and Dubé, 2022).

The Use of Technology in the Montessori Environment

In Montessori education, the use of technology is becoming a topic of growing interest and investigation. Scippo and Ardolino (2021) and Jones (2017) provided valuable perspectives on this matter, examining the impact of technological resources on student learning within the Montessori framework. Scippo and Ardolino (2021) emphasized the need for a harmonious relationship between technological and traditional Montessori materials, while Jones (2017) explored Upper Elementary Montessori teachers' beliefs and practices concerning technology in the classroom. Additionally, Owen et al. (2021) examined the challenges and benefits of incorporating digital literacies into the Montessori Early Childhood curriculum.

Scippo and Ardolino (2021) offered an evidence-based approach to incorporating technological resources into Montessori classrooms. Their study emphasized that integrating technological materials should complement the traditional Montessori materials, fostering a harmonious relationship that encourages creativity, metacognition, and learning (Scippo & Ardolino, 2021). Scippo and Ardolino (2021) findings suggest that interactive software can be integrated effectively into Montessori education, complementing traditional approaches and creating positive learning experiences for students.

Jones (2017) studied Upper Elementary Montessori teachers' beliefs about the use of technology in the classroom and the technological pedagogical content knowledge demonstrated in the technology Montessori teachers choose to use in Lower Elementary school classrooms. This study revealed that Montessori educators, despite expressing positive views about technology and recognizing its importance in developing students' skills, often make software decisions based on accessibility rather than the quality or instructional relevance of the software to classroom content (Jones, 2017). Additionally, while many teachers have the capability to innovate with technology, they may choose not to do so in a Montessori classroom due to time constraints, resource limitations, or a perceived lower significance of these activities (Jones, 2017). These insights prompt contemplation on how to bridge the gap between positive attitudes and practical constraints, ensuring that technology enhances math fluency in the Montessori environment in a manner aligned with its pedagogical principles.

In another study that examined the intersection of the Montessori approach and the use of computer technology, Scippo and Ardolino (2021) studied the effect of computer software on the development of computation thinking, creativity, sense of belonging and self-efficacy in a primary Montessori school. This study demonstrated that incorporating technological materials not only enhances student creativity but also contributes to an increased sense of community (Scippo &

Ardolino, 2021). Additionally, the coexistence of technological resources and traditional developmental materials goes beyond mere presence, leading to integration and a harmonious blend that fosters virtuous cycles of creativity, metacognition, and learning (Scippo & Ardolino, 2021). In light of Scippo and Ardolino's (2021) exploration of the synergy between the Montessori approach and computer technology, their findings resonate with the potential of technology to elevate math fluency in the Montessori environment. This study not only underscores the positive impact of incorporating technological materials on creativity but also emphasizes the enhancement of a sense of community (Scippo & Ardolino, 2021). Moreover, the seamless integration of technological resources alongside traditional developmental materials, as highlighted in this research, suggests a promising avenue for fostering virtuous cycles of creativity, metacognition, and learning in the realm of Montessori math education (Scippo & Ardolino, 2021).

A study by Owen et al. (2021) looked further into the integration of computer technology in the Montessori environment by examining the potential benefits of incorporating focused teaching of digital literacies. The study revealed the importance of explicit guidance that recognizes digital technology as a normal part of a child's social development, requiring active engagement rather than restriction (Owen et al., 2021). The study also highlighted the significance of defining digital literacies within the context of the school, enabling educators to establish a shared language for digital literacies and technology use (Owen et al., 2021). Owen et al. (2021) noted that this shared language provided continuity and compatibility within microsystems, fostering mutual support. The findings highlight the significance of explicit guidance recognizing digital technology as a normal aspect of a child's social development, emphasizing active engagement over restriction (Owen et al.,2021). Establishing a shared language for digital literacies within the school context fosters continuity and compatibility, enabling educators to confidently introduce digital technologies in a manner consistent with the school's philosophy and curriculum (Owen et al., 2021). These insights

resonate deeply with the endeavor to introduce digital technologies in a manner that not only aligns with Montessori principles but also augments the learning experience, particularly in the realm of math fluency.

Conclusion

This literature review underscores the importance of exploring outcomes and potential challenges related to the traditional Montessori approach to math fluency and the integration of technology in the Montessori environment. The studies reviewed collectively reveal the significance of addressing fluency challenges in the transition from Montessori to traditional education, the potential effectiveness of Montessori methods for reading but ambiguity in math outcomes, and the need for a standardized approach to Montessori math materials. Moreover, the review emphasizes the positive impact of interactive software, serious games, and technology-enhanced learning on math fluency, advocating for a balanced integration of technology with traditional Montessori materials. The findings also highlight educators' positive attitudes toward technology but reveal challenges in aligning software choices with instructional relevance. Additionally, the study by Owen et al. (2021) emphasized the need for a common approach, authentic integration, and a shared language for digital literacies in Montessori Early Childhood Education. The identified gaps in knowledge suggest the necessity for further research, particularly in understanding the nuanced impact of Montessori methods on math fluency and the optimal integration of technology in the Montessori environment. This study will explore these gaps by conducting confidential interviews with elementary Montessori educators to collect insights regarding their perspectives on the utilization of Montessori materials for math fluency, offering valuable insights for educators and researchers in the field of Montessori education.

Methods

Research Question

"How do elementary Montessori educators perceive the effectiveness of Montessori materials in enhancing math fluency among students?"

Design

This study utilized a qualitative action research design to investigate the impact of Montessori math materials on math fluency among students. The qualitative action research methodology was deemed appropriate for this study as it allows for an in-depth exploration of participants' experiences and perceptions. According to Merriam (2009), this methodology has been successfully employed in similar educational contexts, providing rich insights into teaching practices and learning outcomes. By adopting this approach, the study aims to effectively address the research question by gaining a nuanced understanding of the experiences and perspectives of Montessori teachers regarding the use of math materials.

Data Collection

The data was collected between January 2024 and March 2024 and consisted of the following two main steps to investigate the research question:

Step 1

Anonymous Teacher Interviews (20-30 minutes each): One-time semi-structured interviews with participating Montessori teachers were conducted to gather insights into their experiences with Montessori math materials, teaching methods, and perceived strengths or weaknesses of the materials.

Step 2

Review of Interview Notes: Participants reviewed the interview notes for accuracy after the interview. The data collected during the interviews was recorded in a written form.

Participants and Setting

The participants for this study were drawn from Montessori teachers actively engaged in teaching mathematics during the data collection period from January 2024 to March 2024. The study included a convenience sample of Lower Elementary-trained Montessori teachers sourced from the Montessori School of Durham and the American Montessori Society (AMS) research participant database. The interviews were conducted remotely via Zoom to ensure anonymity and convenience for the participants. Data was gathered from a total of seven Montessori teachers within the specified time frame. This sample was chosen due to its relevance to the research question, aiming to capture diverse perspectives on the use of Montessori math materials and their impact on math fluency. The study was conducted without any disruptions to the regular teaching activities of the participants, ensuring a naturalistic exploration of their experiences.

Data Analysis

A thematic analysis was conducted to provide a basic understanding of the dataset's characteristics. Before conducting the main analysis, preliminary steps were taken to ensure data quality, including a review of the recorded transcript alongside written notes to confirm accuracy. A multimodal large language model (GPT-4) was then used to summarize and extract themes from each interview. These individual summaries were then synthesized into an overview of key themes which emerged from the data set. The results from the analysis are presented in the subsequent sections, providing insights into the impact of Montessori math materials on math fluency in elementary classrooms.

Findings

An initial exploration of the interview data revealed some overarching patterns and themes, indicating a nuanced understanding of how Montessori math materials contribute to math fluency in elementary classrooms. This was further enriched by the prevalence of non-Montessori materials used and novel approaches to address specific challenges and enhance the learning experience.

Effectiveness of Montessori Math Materials

Out of the seven interviewed teachers, five expressed a positive view on the effectiveness of traditional Montessori math materials for building math fluency, highlighting their utility in conveying abstract concepts and fostering deep understanding. One teacher adopted a notably critical stance, suggesting a complete overhaul of the Montessori math curriculum to better align with current research and encourage a deeper understanding beyond procedural learning. Meanwhile, another teacher provided a nuanced view, appreciating the materials' potential but recognizing the need for supplementation to address areas such as retention of student focus and the teaching of difficult concepts. Overall, the interviews reveal a multifaceted view of the use and effectiveness of Montessori math materials for building math fluency in elementary classrooms. Teachers voiced an appreciation for the tactile and manipulative nature of Montessori materials for conceptual understanding, with several affirming their effectiveness in teaching abstract math concepts. However, there was a noticeable exploration of supplementary materials and non-traditional approaches to address limitations and enhance the curriculum.

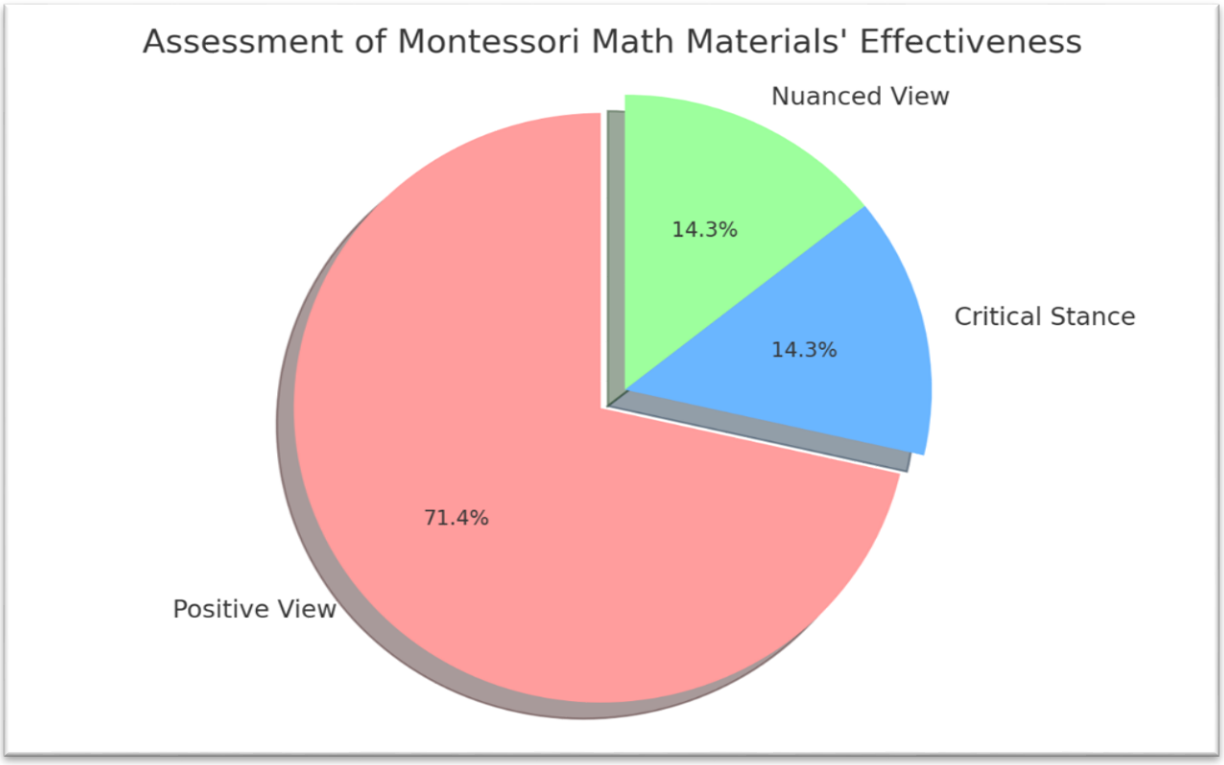

Figure 1: Assessment of Montessori Math Materials' Effectiveness Among Seven Teachers.

Traditional Montessori Materials Used for Math Fluency

Traditional Montessori materials were commonly cited by educators for their effectiveness in enhancing math fluency among elementary students. Among the materials, bead chains, bead bars, and finger charts were most frequently mentioned, each highlighted by teachers in three separate instances. This indicated a strong preference for these tools in fostering an understanding of mathematical concepts through tactile and visual means. Additionally, materials such as addition/subtraction boards, multiplication boards, checkerboards, and pegboards were also noted multiple times, underscoring their integral role in the Montessori math curriculum. Less frequently mentioned, but still notable for their specific uses, were Tables A and B, flat bead frames, decimal system materials, and racks and tubes, each brought up in one instance. The varied use of Montessori materials reflects the flexibility and responsiveness of Montessori

education to individual learning needs and highlights the dynamic nature of teacher practices within this educational framework.

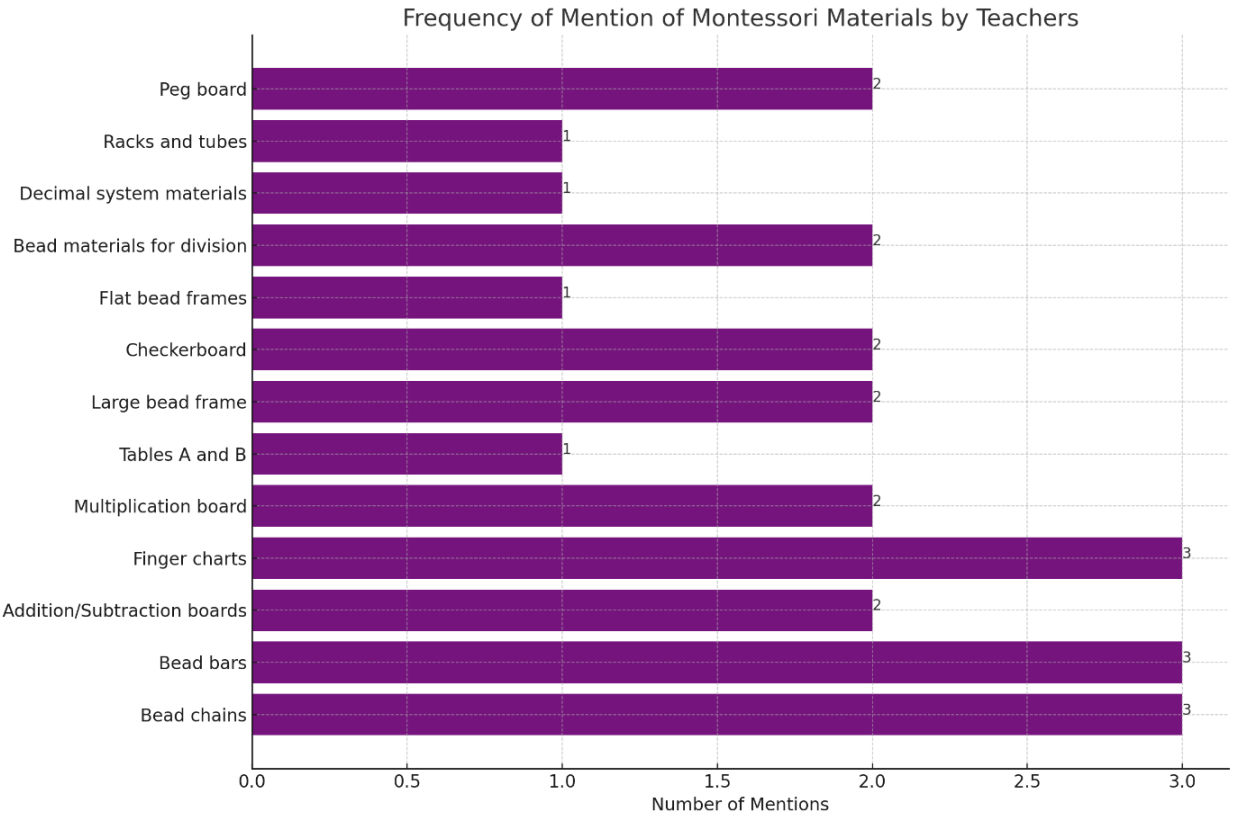

Figure 2. Frequency of Mention of Montessori Materials by Teachers in Interviews.

Challenges with Montessori Materials

Despite the overall effectiveness of the traditional Montessori math materials, some challenges were noted, including the need for repetition and difficulty addressing specific concepts like subtraction and division. One teacher specifically criticized the materials for encouraging a counting strategy over a deeper understanding of numbers, suggesting a disconnect between the materials and current mathematical pedagogy.

Limited Engagement for Older Students

Traditional materials like addition and subtraction strip boards are less engaging for older students, who may require more stimulating activities that challenge their developing math skills.

Difficulty with Abstract Thinking Transition

There is a noted challenge in transitioning students from concrete manipulatives to abstract thinking and mental calculation. This transition is not always naturally developed, which can hinder math fluency as students progress in their education.

Dependence on Materials

Students may develop a dependence on physical materials, which can impede their ability to perform calculations mentally. This reliance can particularly affect their performance in non-Montessori environments where such materials are not used.

Integration with New Technologies

There can be confusion and a lack of effectiveness when integrating new technologies or non-Montessori educational tools such as Khan Academy. This integration often does not align well with Montessori principles, leading to discontinuation and missed opportunities for enhancing learning through technology.

Challenges in Teaching Advanced Math Concepts

Some materials do not adequately support the teaching of more complex mathematical operations or concepts that require a higher level of abstract thinking, such as long division or fractions.

Cost and Accessibility

Financial constraints can limit the availability of Montessori materials for some schools, impacting the uniformity and quality of math education provided to students.

Consistency with Current Research

Some criticisms are directed at the traditional Montessori methods not aligning well with contemporary research in mathematics education, which suggests a need for updates or integration of new methodologies to enhance math fluency and understanding.

Inclusivity Issues

Children with specific learning needs or physical challenges may find it difficult to manipulate the small components of traditional Montessori materials, which could hinder their engagement and learning progress.

Repetitive Nature Requirement

The effectiveness of Montessori materials often relies on continuous repetition, which may not always be appealing or effective for all students, particularly as they grow older and seek more varied educational experiences.

Difficulty in Supplementing with External Programs

While some teachers attempt to integrate external programs or materials to supplement Montessori education, these often do not align well with Montessori principles or confuse students, leading to their discontinuation.

These points highlight the areas where traditional Montessori math materials might be reconsidered or supplemented to better support the diverse needs of students and incorporate modern educational insights.

Supplementary Materials and Approaches

To address these challenges and enhance the learning experience, several non-Montessori materials and approaches were mentioned. These included the use of flashcards and timed sheets for multiplication practice, number bonds, and operations cards to engage students in understanding mathematical operations and word problems. There was an openness to incorporating new trends and innovations, ensuring these align with Montessori principles.

Non-Montessori Materials Mentioned

- Multiplication flashcards
- Madman sheets (leveled multiplication fact sheets with a timer)
- Number bonds (inspired by public education trends)
- ETC Montessori math operations cards with word problems and graphing exercises
- Dice games and MOBY (a math crossword game)
- Calculators for checking answers
- Supplemental decanomial map

Programs outside the classroom

- Kumon Tutoring
- Russian Math Tutoring
- Khan Academy Online Tutoring

Non-Traditional Approaches Mentioned

- Making materials more game-like for socialization and repetition
- Incorporating various games to address attention and occupational therapy challenges
- Using calculators for answer verification
- Exploring current research for new ideas in mental math improvement.
- Encouraging student-driven strategies and coding, robotics integration for older students.

Assessment and Student Outcomes

Teachers explained that they assess math fluency through observation, practical application, and informal methods rather than standardized testing. Significant student outcomes include mastering multiplication facts, understanding the connection between mathematical operations, and achieving fluency through engagement with Montessori materials.

Final Assessment on Traditional Montessori Math Materials

The majority of teachers (five out of seven) find Montessori math materials effective for building math fluency, highlighting their ability to teach abstract concepts in a concrete manner and to adapt to the individual learning needs of students. However, one teacher advocates for a significant overhaul, arguing that they do not align with contemporary educational research on mathematical learning and overly emphasize counting strategies.

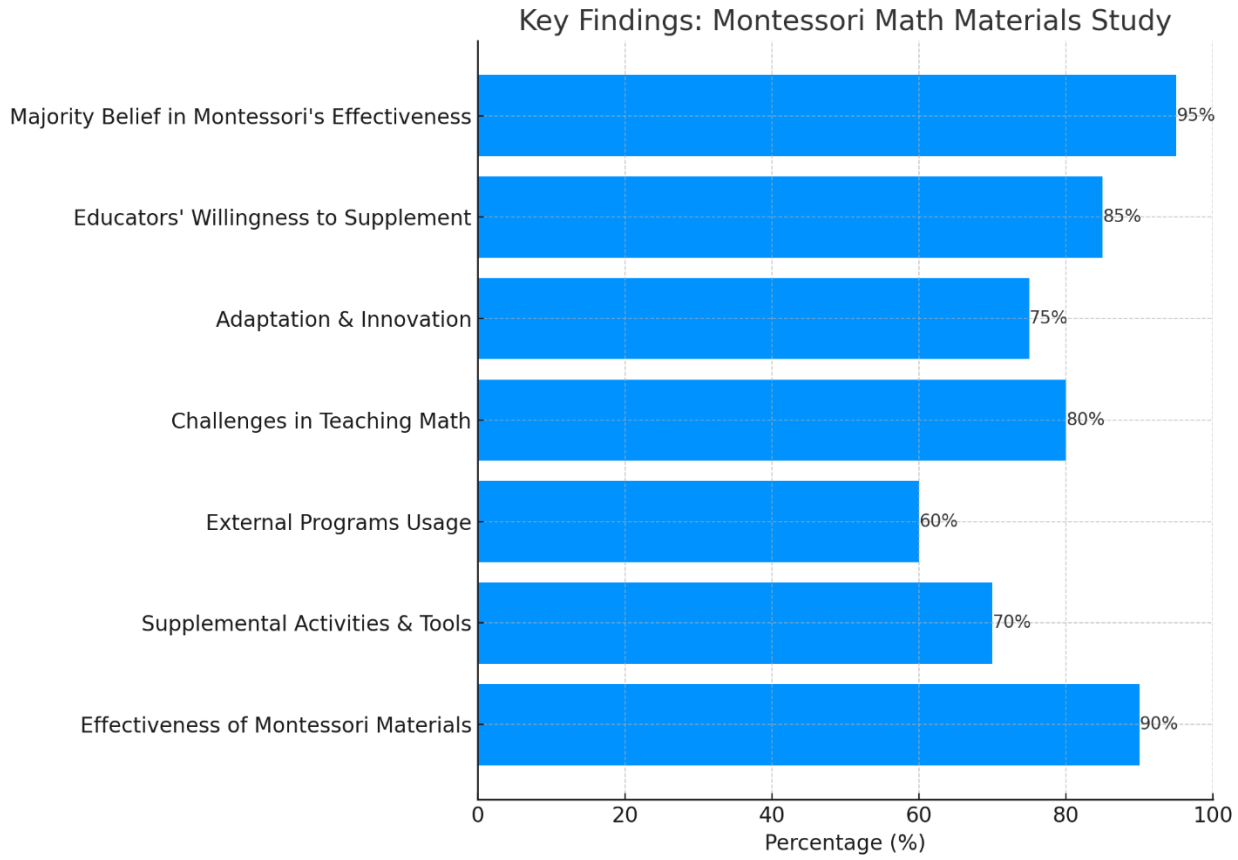

Figure 3. Key Findings from the Study on Montessori Math Materials for Increasing Fluency.

Discussion

The findings from this study offer several insights into the effects of traditional Montessori math materials on math fluency. This section highlights interpretations, comparisons with existing literature, and broader implications of these results.

Interpretation of Key Findings

The interviews collectively suggest that while Montessori math materials are foundational to building math fluency in elementary students, there is room for improvement and supplementation. These insights point towards an evolving Montessori practice that remains open to innovation and adaptation, ensuring the methodology continues to meet the needs of students in a changing educational landscape. The findings reveal a complex picture of Montessori math education that is largely positive but acknowledges areas for growth. Educators are generally supportive of Montessori math materials for their effectiveness in building math fluency but also advocate for the incorporation of supplementary materials and approaches to address gaps and enhance the learning experience. This adaptability and openness to innovation suggests a dynamic approach to Montessori education, aiming to maintain its core principles while evolving to meet the needs of today's students.

Comparison with Existing Literature

Utilizing manipulatives and concrete materials for teaching math resonates with the findings of Basargekar and Lillard (2021), where the Montessori approach's emphasis on hands-on, experiential learning is highlighted. Both sets of findings underscore the value of concrete materials in developing a deep understanding of mathematical concepts, moving from concrete to abstract learning. The interview data, emphasizing the effectiveness of Montessori math materials in building concrete understandings, align with Basargekar and Lillard's observations that Montessori students show a stronger grasp of deeper conceptual knowledge as opposed to procedural fluency.

The challenges noted with Montessori materials, particularly in addressing specific concepts like subtraction and division, and the criticism of encouraging a counting strategy, reflect the concerns highlighted by Snyder et al. (2021) and Carbonneau et al. (2013) regarding the potential limitations of manipulatives and Montessori methods in improving standardized test proficiency and mathematical fluency. Both sets of insights suggest the necessity for a nuanced application of Montessori principles, considering instructional variables and the role of teacher-guided instruction, especially for students with mathematical difficulties.

The openness to incorporating non-Montessori materials and technological innovations aligns with the positive outlook on integrating interactive software and technology-enhanced learning platforms noted by Es-Sajjade & Paas (2020) and Kurvinen et al. (2020). This comparison underscores a shared recognition of the benefits of blending traditional Montessori materials with modern educational tools and approaches, including serious games and digital literacies, to enhance math fluency and engagement.

The method of assessing math fluency through observation and practical application in Montessori classrooms mirrors the findings by Brown & Lewis (2016) and Basargekar and Lillard (2021) that Montessori students may not perform as well on standardized tests but exhibit a strong conceptual understanding of mathematics. This similarity highlights the difference in assessment approaches and outcomes between Montessori and traditional educational settings, reinforcing the need for further research into effective evaluation methods that capture the breadth of Montessori student achievements.

In sum, the initial exploration of interview data regarding the use and perceptions of Montessori math materials complements existing literature on Montessori education's effectiveness in math fluency. It shows the practical application of Montessori principles in classrooms, educators' adaptive strategies to enhance learning experiences, and the nuanced

impact of Montessori methods on math fluency. This comparison underscores the ongoing dialogue between traditional Montessori approaches and contemporary educational practices, highlighting the potential for positive educational change through the integration of technology and innovation within the Montessori framework.

Implications

The implications of this study extend significantly into educational practice, pedagogical theory, and future research directions. At the core of these implications is the recognition of the enduring value of Montessori math materials in fostering a concrete understanding of mathematical concepts, which is crucial for the development of math fluency in elementary students. This study corroborates existing literature by demonstrating that Montessori materials effectively bridge concrete and abstract mathematical understanding through hands-on experiences, tailored learning pathways, and promotion of student autonomy.

However, the study also highlights critical areas for enhancement within the Montessori mathematical framework. The identification of challenges, such as the need for repetition and the difficulty in addressing specific mathematical concepts, points towards the necessity for the incorporation of supplementary materials and methodologies. This suggests a pivotal implication for Montessori educators and curriculum developers: there is a clear benefit in integrating innovative tools and approaches, such as interactive software and digital resources, to complement traditional Montessori pedagogy. The study illustrates a growing openness among Montessori educators to embrace such integrations, aiming to align with foundational Montessori principles while enhancing engagement and applicability of math concepts in real-world contexts.

For educational practitioners, this study serves as a call to action to critically evaluate and expand their instructional strategies to incorporate both traditional and innovative resources. The positive reception towards non-Montessori materials and approaches among Montessori

educators underscores the potential for a more inclusive, adaptive teaching model that retains the core advantages of Montessori education while addressing its limitations. This adaptive approach is not only relevant to Montessori settings but also offers valuable insights for broader educational contexts, suggesting that a blend of hands-on, experiential learning with digital innovations can enhance math fluency across diverse learning environments.

The study also lays the groundwork for future research in several ways. First, it highlights the need for further empirical studies to explore the long-term impacts of integrating digital tools within Montessori math education on students' math fluency and overall academic achievement. Second, it calls for research into the development of interactive software and digital resources that are specifically designed to complement Montessori pedagogy, ensuring that these tools align with the Montessori principles of self-directed learning, exploration, and discovery. Lastly, the study underscores the importance of investigating the effects of such integrations across diverse educational settings and student populations, contributing to a more inclusive and comprehensive understanding of effective math fluency development strategies. The implications of this study are multifaceted, addressing educators, curriculum developers, and researchers alike. By acknowledging the strengths of Montessori math materials while recognizing the value of supplementary digital innovations, this research paves the way for an enriched educational approach that is both reflective of Montessori's enduring pedagogical insights and responsive to the demands of the 21st-century learning landscape.

Limitations and Future Research

This study, while providing valuable insights into the impact of Montessori math materials on math fluency, is subject to several limitations that should be considered when interpreting the findings. The qualitative nature of the study, focusing on interviews with a limited number of Montessori educators, may not fully capture the diversity of experiences and perspectives across the broader Montessori community. The sample size and selection process, relying on convenience sampling from specific Montessori schools, and groups, potentially limit the generalizability of the findings to all Montessori educational settings. Secondly, the study's reliance on educator reports without direct observation or assessment of student outcomes means that the reported effectiveness of Montessori materials and the integration of supplementary tools are not empirically verified through student performance data. The subjective nature of self-reported data may also introduce bias, as educators may have varying interpretations of what constitutes math fluency and the effectiveness of certain materials or methods. Furthermore, the study did not extensively explore the potential variability in the implementation of Montessori methods and the use of non-Montessori materials across classrooms, which could significantly affect the outcomes. The degree to which these materials are integrated, and the fidelity to Montessori principles in such integration, could vary widely and impact the effectiveness of these tools in promoting math fluency. Lastly, the study's focus on elementary Montessori education means the findings may not be directly applicable to older or younger student populations within the Montessori system. The developmental stage of students can significantly influence the effectiveness of educational materials and strategies, suggesting a need for caution in applying these findings across all age groups.

Future Research

Given the limitations identified, several avenues for future research emerge. Future studies could employ a mixed-methods approach, combining qualitative insights from educators with quantitative data on student performance, to provide a more comprehensive understanding of the impact of Montessori math materials on math fluency. This approach would allow for the correlation of educator perceptions with actual student outcomes, offering a more objective measure of effectiveness.

Further research could also explore a broader and more diverse sample of Montessori schools, including those in different geographical regions and serving diverse student populations. Such studies could help assess the generalizability of the findings and explore how contextual factors influence the integration and effectiveness of Montessori and supplementary materials. Investigations into the implementation fidelity of Montessori principles when integrating non-Montessori materials and the role of teacher training in this process would provide deeper insights into how these innovations can be effectively harmonized with traditional Montessori pedagogy. Research in this area could guide the development of best practices for incorporating digital tools and other resources in Montessori classrooms. Additionally, longitudinal studies examining the long-term impacts of using Montessori materials and supplementary digital tools on math fluency would be valuable. Such research could track student progress over time to determine the sustained effects of these educational strategies on math skills development.

Conclusion

This study contributes to the body of knowledge on the impact of Montessori math materials on math fluency in elementary education. The findings underscore the effectiveness of traditional Montessori materials in building foundational math skills through hands-on, tactile learning experiences. At the same time, the research highlights the potential for these materials to

be supplemented with additional tools and approaches to address specific challenges and enhance learning outcomes. Despite the positive reception of Montessori materials among educators, the study revealed areas for improvement, particularly in integrating modern educational technologies and supplementary resources. This paves the way for future pedagogical enhancements that could make Montessori education more responsive to the diverse needs of today's students. The openness of educators to such integration suggests a dynamic evolution of the Montessori approach, aiming to retain its core principles while adapting to contemporary educational demands. Furthermore, the implications of this study are manifold, affecting educational practice, pedagogical theory, and future research directions. For practitioners, the findings serve as a call to critically evaluate and expand instructional strategies to include a blend of traditional and innovative resources, thereby fostering a more inclusive and adaptive teaching model. Lastly, this research lays the groundwork for future studies to explore the integration of digital resources in Montessori education further, investigate the long-term impacts of these integrations on student outcomes, and broaden the scope to include more diverse educational settings. The study's insights not only reaffirm the value of Montessori materials in fostering math fluency but also highlight the importance of continual innovation in educational practices to meet the evolving needs of students across various learning environments.

References

Basargekar, A., & Lillard, A. S. (2021). Math achievement outcomes associated with Montessori education. *Early Child Development and Care*, *191*(7–8), 1207–1218. https://doi.org/10.1080/03004430.2020.1860955

Baker, A. T., & Cuevas, J. A. (2018). The importance of automaticity development in mathematics. *Georgia Educational Researcher*, *14*(2). https://doi.org/10.20429/ger.2018.140202

Berrett, A. N., & Carter, N. J. (2017). Imagine math facts improves multiplication fact fluency in third-grade students. *Journal of Behavioral Education*, 27(2), 223–239. https://doi.org/10.1007/s10864-017-9288-1

Brown, K., & Lewis, C. W. (2016, November 30). A comparison of reading and math achievement for African American third-grade students in Montessori and other magnet schools. *Journal of Negro Education*. https://eric.ed.gov/?q=a&pg=7751&id=EJ1214088

Cumming, J. J., & Elkins, J. (1999). Lack of automaticity in the basic addition facts as a characteristic of arithmetic learning problems and instructional needs. Mathematical Cognition, 5(2), 149-180. doi:10.1080/135467999387289

Es-Sajjade, A., & Paas, F. (2020). Educational theories and computer game design: Lessons from an experiment in elementary mathematics education. *Educational Technology Research and Development*, 68(5), 2685–2703. https://doi.org/10.1007/s11423-020-09799-w

Fraga-Varela, F., Vila-Couñago, E., & Martínez-Piñeiro, E. (2021). The impact of serious games in mathematics fluency: A study in primary education. *Comunicar, 29*(69), 125–135. https://doi.org/10.3916/c69-2021-10

Gliksman, Y., Berebbi, S., & Henik, A. (2022). Math fluency during Primary School. *Brain Sciences*, 12(3), 371. https://doi.org/10.3390/brainsci12030371

Hurdle, Z. B. (2016, November 30). Aspects that arise in the transition from the Montessori method to a traditional method: A fourth-grade mathematics view. *ProQuest LLC*. https://eric.ed.gov/?id=ED580450

Jones, S. (2017). Technology in the Montessori Classroom: Teachers' beliefs and technology use. *Journal of Montessori Research*, *3*(1), 16. https://doi.org/10.17161/jomr.v3i1.6458

Kacmaz, G., & Dubé, A. K. (2022). Examining pedagogical approaches and types of mathematics knowledge in educational games: A meta-analysis and Critical Review. *Educational Research Review*, 35, 100428. https://doi.org/10.1016/j.edurev.2021.100428

Krath, J., Schürmann, L., & Von Korflesch, H. F. O. (2021). Revealing the theoretical basis of gamification: A systematic review and analysis of theory in research on gamification, serious games and game-based learning. *Computers in Human Behavior*, *125*, 106963. https://doi.org/10.1016/j.chb.2021.106963

Kurvinen, E., Kaila E., Laakso, M.J., & Salakoski, T. (2020). Long term effects on technology-enhanced learning: The use of weekly digital lessons in Mathematics. *Informatics in Education*, 19(1), 51–75. https://doi.org/10.15388/infedu.2020.04

Merriam, S. B. (2009). Qualitative research: A guide to design and implementation. USA: John Wiley & Sons

Morano, S., Randolph, K., Markelz, A. M., & Church, N. (2020). Combining explicit strategy instruction and mastery practice to build arithmetic fact fluency. *TEACHING Exceptional Children*, *53*(1), 60–69. https://doi.org/10.1177/0040059920906455

The National Council of Teachers of Mathematics, Inc., Joyner, J., Reys, B., Silver, E. A., Schoenfeld, A., Lindquist, M. M., Galindo, E., Ferrini-Mundy, J., & Martin, W. G. (2000). *Principles, standards, and for school mathematics*. The National Council of Teachers of Mathematics, Inc.

NAEP Mathematics: National Achievement-Level Results. (n.d.). https://www.nationsreportcard.gov/mathematics/nation/achievement/?grade=8

Owen, S., & Davies, S. (2020). Maintaining an empowered school community: Introducing digital technologies by building digital literacies at Beehive Montessori School. *London Review of Education*, *18*(3). https://doi.org/10.14324/lre.18.3.03

Policies and Recommendations - National Council of Teachers of Mathematics. (n.d.). https://www.nctm.org/Research-and-Advocacy/policies-and-recommendations/

Scippo, S., & Ardolino, F. (2021). Computational thinking in Montessori primary school. *DOAJ (DOAJ: Directory of Open Access Journals)*. https://doi.org/10.6092/issn.1970-2221/12163

Snyder, A., Tong, X., & Lillard, A. S. (2021). Standardized test proficiency in Public Montessori Schools. *Journal of School Choice*, 16(1), 105–135. https://doi.org/10.1080/15582159.2021.1958058

Stickney, E. M., Sharp, L. B., & Kenyon, A. S. (2012). Technology-enhanced assessment of math fact automaticity. *Assessment for Effective Intervention*, *37*(2), 84–94. https://doi.org/10.1177/1534508411430321

Woodward, J. (2006). Developing automaticity in multiplication facts: Integrating strategy instruction with timed practice drills. *Learning Disability Quarterly*, *29*(4), 269–289. https://doi.org/10.2307/30035554

Appendix A

Teacher Interview Questions

Use of Current Math Materials:

a. Can you describe the math materials currently in use for building math fluency in your Montessori classroom?
b. How frequently are these materials used during math instruction?
c. Are there specific lessons or activities where these materials are particularly effective?

Perceived Effectiveness of Existing Materials:

a. In your opinion, how effective are the current math materials for building math fluency?
b. Can you provide examples of student outcomes or improvements attributed to these materials?
c. Are there any limitations or shortcomings you've observed with the current materials?

Specific Challenges in Teaching Math with Montessori Materials:

a. What specific challenges or difficulties do you encounter when using Montessori materials to teach math fluency?
b. Are there any particular concepts or skills that seem to be more challenging for students using these materials?
c. How do you address these challenges in your teaching approach?

Suggestions for Updating or Supplementing Math Materials:

a. Are there any changes or updates you would recommend for the current math materials?
b. Do you believe that supplementing the existing materials with additional resources would be beneficial? If so, what type of materials?
c. How do you envision these updates or supplements improving the math education experience?

Emerging Trends or Innovations in Montessori Math Education:

a. Have you come across any emerging trends or innovative approaches in Montessori math education recently?
b. Are there any new technologies or teaching strategies that you think could enhance math instruction using Montessori principles?
c. How open are you and your colleagues to incorporating these new trends or innovations into your teaching practices?

Overall Reflection on Montessori Math Education:

a. What are the key strengths of Montessori math education in building math fluency?
b. Are there any unique advantages that Montessori materials offer compared to other math teaching methods?