Combining Explicit Strategy Instruction and Mastery Practice to Build Arithmetic Fact Fluency

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TEACHING Exceptional Children, Vol. 53, No. 1, pp. 60–69. Copyright 2020 The Author(s). DOI: 10.1177/0040059920906455

Ms. Kent is the special education teacher in a fourth-grade inclusive classroom. The class recently began working on adding and subtracting fractions with unlike denominators, and Ms. Kent has become concerned about the difficulties several students with disabilities are encountering. In attempting the complex fraction computation procedures, the students with disabilities make many fact errors, mix up or skip procedural steps, and become frustrated before giving up altogether. From her observations, Ms. Kent believes this is caused by students' lack of basic whole-number arithmetic fact fluency. In other words, she suspects that their inability to retrieve math facts from memory quickly and accurately is strongly contributing to the difficulty and frustration they are experiencing with fractions. The students seem to follow Ms. Kent's models, but when they work independently, their lack of multiplication fact knowledge slows them down and interferes with their ability to track procedures and focus on the bigger picture. Mr. Stephens, Ms. Kent's general education co-teacher, agrees that a number of their students need to improve their fact fluency, but the curriculum program adopted by their school does not include systematic fact fluency–building work. In an effort to address their students' needs, he supplements the curriculum with computer-based fluency practice one or two times a week for about 10 minutes. However, these activities do not seem to be having a meaningful impact on struggling students' fact fluency. Ms. Kent believes that building students' fluency will facilitate their learning of more complex mathematics skills and concepts, but she is not sure where to begin.

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Math fact fluency involves the quick, accurate retrieval of basic arithmetic combinations and the ability to use this fact knowledge efficiently (Baroody, 2011). Math fact retrieval is typically considered fluent when performed accurately within 2 to 3 seconds (Burns et al., 2010; Stickney et al., 2012), and *efficiency* refers to students' ability to apply fact knowledge to more complex mathematical skills and concepts. This definition of fact fluency considers both declarative knowledge (i.e., basic fact recall) and conceptual knowledge, or deeper understanding of number concepts and ability to apply this knowledge flexibly. The Common Core State Standards (CCSS) include fluency with addition and subtraction facts within 100 by the end of Grade 2 and with multiplication and division facts within 100 by the end of Grade 3 (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

Fact fluency is crucial for estimation, complex computation, mental calculation, word problem solving, rational number learning, algebra, and overall success in mathematics (National Mathematics Advisory Panel [NMAP], 2008). Given the role it plays in mathematics achievement, fact fluency is an important concern for students with disabilities and difficulties in mathematics, as these groups of students often demonstrate consistent and persistent deficits with fluent retrieval of math facts (Geary, 2004; Geary et al., 2007). Further, automatic fact retrieval is often taken for granted by teachers and in curriculum materials when more complex concepts and procedures are introduced (Woodward, 2006), so students who lack fact fluency can experience high cognitive load and frustration when learning more complex material (Jitendra et al., 2007).

For example, to add fractions with unlike denominators, students must use multiplication fact knowledge to (a) determine the lowest common denominator, (b) convert fractions by multiplying numerators and denominators by a constant, and potentially, (c) simplify the sum. This complex procedure taxes students' information processing capacity (LeFevre et al., 2005). However, fluent fact retrieval can facilitate a student's ability to execute complex procedures accurately because their cognitive resources are not consumed by retrieving facts (Tronsky, 2005). When a student is fluent with arithmetic facts, they have greater capacity for more complex learning, like developing understanding of the concept of equivalence (LeFevre et al., 2005). In contrast, lacking fact fluency (and reliance instead on inefficient counting strategies or a calculator) can lead to fact errors and procedural mistakes as well as failure to understand underlying math concepts (Cumming & Elkins, 1999).

Ms. Kent begins to ask her colleagues for ideas and resources for building fact fluency. Some of her colleagues recommend teaching strategies for deriving facts and emphasize the importance of building number sense. Others recommend traditional drill-based practice and emphasize the importance of developing fast, accurate retrieval from memory. Ms. Kent believes her students need to develop both competencies—number sense and conceptual understanding, and fast, accurate retrieval from memory—to apply their knowledge efficiently to the more complex mathematics skills and concepts they are currently struggling with.

Effective Fluency Instruction

The Institute for Education Sciences (Gersten et al., 2009) recommends that students receive about 10 minutes per day of instruction focused on fluent retrieval of arithmetic facts. In general, there are two approaches to fact fluency instruction: strategy instruction and mastery practice. Strategy instruction teaches efficient methods for deriving facts and highlights patterns to help students organize and generalize their fact knowledge (Baroody et al., 2009; Isaacs & Carroll, 1999). Strategy instruction can teach basic rules or properties (e.g., the add-1 rule; Purpura et al., 2016) or can focus on patterns like doubles (Van de Walle et al., 2018). Strategy instruction also often incorporates graphic organizers or visual representations, like fact-family diagrams, 10 frames, and number lines (Van de Walle et al., 2018).

Mastery practice, in contrast, resembles traditional drill-type practice

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and typically involves discrete learning trials and many practice opportunities with carefully selected sets of facts to build students' speed and accuracy in responding (Clarke et al., 2016). Mastery-practice sets are often individualized and include a specific ratio of known to unknown facts (Burns et al., 2019). Mastery practice can be implemented using a variety of formats, including flashcard practice, computer-based practice, partner practice, and more.

Several studies have compared the effects of the two approaches, and there is mixed evidence on which approach is most effective. Research suggests that mastery practice is likely more effective for improving speed and accuracy in fact retrieval (i.e., digits correct per minute on math fact probes; Kanive et al., 2014; Poncy et al., 2010). However, strategy instruction may better support application and generalization of fact knowledge to more complex problems (i.e., the ability to apply fact knowledge to multidigit computation problems or word problems; Kanive et al., 2014; Woodward, 2006). Current standards require both automaticity with facts and the ability to apply fact knowledge flexibly to solve problems (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), and the National Council for Teachers of Mathematics (2000) argues that "developing fluency requires a balance and connection between conceptual understanding and computational proficiency" (p. 35).

Studies have found that a combined approach to fluency instruction, or the practice of using both explicit strategy instruction and mastery practice, can improve both automaticity and generalization. Ok and Bryant (2016) tested the effects of a combined approach to fact fluency instruction (i.e., strategy instruction + mastery practice), and Woodward (2006) tested the effects of a combined approach against mastery practice alone. In both cases, researchers found that all students participating in a combined approach made significant gains in their fact fluency. In Woodward's study, students who participated in the combined approach also achieved greater gains in their ability to generalize their fact knowledge to more complex computation problems (i.e., 2 × one-digit problems) and to mental calculation (e.g., round to mentally solve 3 × one-digit computation problem). Given that the evidence suggests that using a combined approach is effective for building both fluency and generalization, a combined approach may be preferable to using either method (strategy instruction or mastery practice) in isolation.

Theoretical support for combining the approaches comes from the active construction (Baroody et al., 2009) view of fact learning. According to the active construction view or number sense view of fact learning (Gersten & Chard, 1999), fact fluency grows out of number sense because developing fluency relies on a connected body of knowledge that involves patterns, relations, algebraic rules, and reasoning processes as well as facts. Baroody et al. (2009) provide the following examples:

> Consider two examples. Once children recognize that addition with zero does not change a number, this pattern can be stored as the algebraic rule: "the sum of any number and zero is that number" ($n + 0/0 + n = n$ rule). Knowing that a number in the counting sequences is 1 more than the previous number (e.g., "four" is one more than "three") can enable a child to use a known "doubles" combination to logically deduce the sum of a "near double" (e.g., If $3 + 3 = 6$, and 4 is 1 more than 3, then the sum of $3 + 4$ must be 1 more than 6, which is 7). (p. 70)

Combining strategy instruction and mastery practice addresses conceptual and factual-level learning in a mutually supportive way. Whereas teaching strategies provides students with efficient methods for deriving facts and helps them recognize patterns among facts (i.e., develop number sense), providing mastery practice strengthens memory and builds students' capacity to retrieve facts quickly and accurately (Burns, 2005; Codding et al., 2007).

The following sections of this article describe how to implement explicit fact strategy instruction and a variety of evidence-based mastery-practice activities. Following the sections on each type of activity, there is a section that provides guidance from the research about how to successfully combine the two methods to build students' fact fluency.

After further reading and reflection about fact fluency instruction, Ms. Kent confers with Mr. Stephens about the possibility of devoting a short block of time each day to fluency-building activities. They compromise and decide to use 15 minutes three times per week for fluency instruction and practice. Ms. Kent recommends that they combine explicit strategy instruction with a variety of mastery-practice activities. She thinks that mastery practice will help students retrieve facts more quickly and accurately and that teaching strategies will help build students' number knowledge.

Explicit Strategy Instruction

Explicit strategy instruction is an effective approach for teaching various mathematics skills and concepts to students with and without disabilities (e.g., Gersten et al., 2009) and can be used to help students with disabilities become more strategic learners (Luke, 2006). In addition, learning strategies for deriving facts can support long-term retention (Woodward, 2006) and can provide students with tools they can use when they are unable to retrieve facts from memory. Explicit strategy instruction includes clear, direct explanations of strategy use and task performance, teacher modeling of the strategy procedures with think-aloud, and scaffolded prompts and questioning designed to gradually release responsibility for independent strategy selection and use to students (e.g., Swanson, 1999).

Table 1 **Multiplication Fact Strategies**

Fact set	*Strategy or rule*	*Example*
Easier facts and strategies: Based on rules or counting patterns		
Times 0	Rule: Any number multiplied by 0 equals 0 (i.e., the zero property of multiplication) Strategy: If one of the factors is 0, the product is 0.	$0 \times 9 = 0$
Times 1	Rule: Any number multiplied by 1 equals itself (i.e., the identity property of multiplication) Strategy: If one of the factors is 1, the product is the other factor.	$1 \times 9 = 9$
Times 10	Strategy: Find the product of a "times 10" fact by counting by 10s the number of times indicated by the other factor.	$10 \times 9 = 90$
Times 5	Strategy: Find the product of a "times 5" fact by counting by 5s the number of times indicated by the other factor.	$5 \times 9 = 45$
Times 2: Doubles	Rule: 2 multiplied by any number equals the number added to itself. Strategy: To multiply any number by 2, double it.	$9 \times 2 = 18$
More challenging facts and strategies: Derived or based on patterns		
Perfect squares or times itself	Rule: When both factors are the same, the product will be a perfect square. Strategy: A number times itself will form a square array.	$9 \times 9 = 81$
Times 9	Pattern 1: Starting at 54, the products of the "times 9" facts flip over and mirror previous products. Pattern 2: The 10s digit of the product of a "times 9" fact is always 1 less than the other factor. Pattern 3: The sum of the two digits in the product of a "times 9" fact is always 9. Strategy: Times 10 minus 1. Replace 9 with 10, then subtract the other factor from the product.	Pattern 1: 54 is the mirror of 45; 63 is the mirror of 36, etc. Pattern 2: $9 \times 6 = 54$; and 5 is 1 less than 6. Pattern 3: $9 \times 7 = 63$; and $6 + 3 = 9$. Strategy: $7 \times 9 = 7 \times 10 - 7$.
Times 3: Helping facts— doubles plus 1	Strategy: To multiply any number by 3, double it and then add one more set of that factor.	$8 \times 3 = 8 \times 2 + 8$
Times 6: Helping facts—times 5 plus 1	Strategy: To multiply any number by 6, start with "times 5," then add one more set of that factor.	$6 \times 7 = 5 x 7 + 7$
Times 4: Double-doubles	Strategy: To multiply any number by 4, double the number and then double that product.	$6 \times 4 = 6 \times 2 = 12$ and $12 \times 2 = 24$
Times 8: Double-double-doubles	Strategy: To multiply any number by 8, double the number, then double the product, and finally, double that product.	$6 \times 8 = 6 \times 2 = 12$; $12 \times 2 = 24$; and $24 \times 2 = 48$
Helping facts: Add one more set *Use for any unknown fact	Strategy: Start with a fact you know, then add one more set.	$6 \times 7 = 5 \times 7 + 7$
Half then double *Use for any unknown facts with an even factor	Strategy: Select the even factor and divide it in half. Multiply the half factor by the other factor, then double the product.	$6 \times 7 = (3 \times 7) + (3 \times 7)$ or $6 \times 7 = (3 \times 7) \times 2$

Note. Compiled and adapted from French (2005), Miller et al. (1996), Van de Walle et al. (2018), Wood and Frank (2000), and Woodward (2006).

Figure 1 Example explicit fact strategy minilesson

'Times 2' Strategy Mini-Lesson

Teacher Model

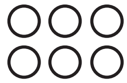
Say: Watch and listen as I show you how to use the 'times 2' multiplication fact strategy. When we multiply by 2, we *repeat* the other factor.

Ask: What does it mean to repeat?
Elicit responses and come to an understanding that *repeat* means to do something again.

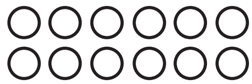
Say: So, when we multiply by 2, we repeat or make a copy of the other factor.

Write and say: Here are some 'times 2' facts: 2 x 3 2 x 6 2 x 9

Say: To use the 'times 2' strategy to solve the first fact, 2 x 3, I can make an array. I'll start by making one set of 3 circles (draw the set), then repeat the set (draw another set of circles right underneath the first set). Now I have 2 sets of 3 circles, or 6 total circles. 2 x 3 = 6.



Say: To use the 'times 2' strategy to solve the next fact, 2 x 6, I'll make another array. I'll start by making one set of 6 circles (draw the set), then repeat the set (draw another set of 6 circles right underneath the first set).
Ask: How many sets of 6 circles do I have? 2 sets.
Ask: And how many total circles do I have? 12.
Ask: So what is 2 x 6? 2 x 6 = 12.

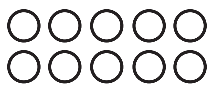


Model the same process with additional 'times 2' facts as needed.

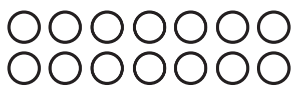
Guided Practice

Say: Now, let's practice making arrays to show 'times 2' facts together.

Say: For this first fact, 2 x 5, follow along with me on your whiteboard. When we multiply by 2, we repeat the other factor. So let's start with the other factor-- 5. Draw a set of 5 circles. Now, to multiply by 2, we repeat, so let's draw another set of 5 circles right underneath the first set. Now, we have 2 sets of 5 circles.
Ask: How many circles do we have in all? 10
Ask: So what is 2 x 5? 2 x 5 = 10



Say: Our next fact is 2 x 7.
Ask: What happens when we multiply by 2? We repeat the other factor.
Ask: How can I make an array to show 2 x 7? Make one set of 7 circles, then repeat the set.
Say: Go ahead and make that array.
Ask: How many circles are in your array? 14
Ask: So what is 2 x 7? 2 x 7 = 14



Say: For this last fact, 2 x 2, remember the 'times 2 strategy' and solve the fact.
Ask: What is 2 x 2? 2 x 2 = 4

Say: Now we're going to wrap up by using the 'times 2' strategy with and without arrays to solve facts on your own.

Monitor students closely and provide feedback as they solve 3 'times 2' facts with arrays and 10 'times 2' facts without arrays.

Fact strategies are intended to teach students efficient strategies for deriving facts, to highlight rules and patterns that can support memory, and to help students develop a connected body of number knowledge. Research indicates that some facts and strategies are easier for students and some are more challenging (Woodward, 2006). ***Table 1*** provides a possible instructional sequence of easier-to-more-complex multiplication fact strategies. Teachers can initially present a fact strategy in a brief, explicit minilesson by modeling how to apply the strategy and providing students with scaffolded practice

Table 2 **Mastery-Practice Activity Implementation Resources**

Mastery-practice activity	*Practice guides*	*Video models*	*Sample materials*
Cover, copy, compare (CCC)	InterventionCentral.org CCC practice guide: https://www.interventioncentral.org/sites/default/files/pdfs/pdfs_interventions/cover_copy_compare_math_facts_revised.pdf	InterventionCentral.org CCC video model: https://www.youtube.com/watch?v=QYU-70ajZhM	InterventionCentral.org CCC worksheet template: https://www.interventioncentral.org/sites/default/files/pdfs/pdfs_interventions/ccc_worksheet_spelling_sight_words_math_horizontal.pdf
Incremental rehearsal	Evidence Based Intervention Network intervention brief: http://ebi.missouri.edu/wp-content/uploads/2011/03/Incremental-Rehearsal-Intervention-Brief-2.pdf InterventionCentral.org incremental-rehearsal practice guide: https://www.interventioncentral.org/academic-interventions/math-facts/math-computation-promote-mastery-math-facts-through-incremental-re	Evidence Based Intervention Network incremental-rehearsal video model: https://www.youtube.com/embed/ke4HETehE6Q?rel=0	VarsityTutors.com free flashcard creator: https://www.varsitytutors.com/aplusmath/flashcards
Constant time delay (CTD) and progressive time delay	InterventionCentral.org CTD practice guide: https://www.interventioncentral.org/sites/default/files/pdfs/pdfs_blog/Instruction_constant_time_delay.pdf Evidence Based Instructional Practices progressive time delay resource: https://ebip.vkcsites.org/progressive-time-delay/	Progressive time delay video models: https://www.youtube.com/watch?v=uRoHUPH1_pk https://www.youtube.com/watch?v=U7DbT0o7fUE	InterventionCentral.org CTD recording sheet: https://www.interventioncentral.org/sites/default/files/pdfs/pdfs_blog/Instruction_constant_time_delay_2.pdf
Taped problems	New York State Response to Intervention Project taped problems intervention guide: https://nysrti.org/intervention-tools/math-tools/tool:recorded-problems/ Evidence Based Intervention Network taped problems intervention brief: http://www.interventionexpress.com/uploads/1/6/8/5/16851140/taped-problems.pdf	Taped problems video model from ElmbrookMTSS: https://www.youtube.com/watch?v=f7xum6xZOtE	Audiovoicerecorder.com free audio recorder for creating MP3 files/taped problem audio files: https://audiovoicerecorder.com/

opportunities to use the strategy with feedback. An example explicit strategy minilesson is presented in ***Figure 1***.

Mastery-Practice Activities

After teaching a fact strategy, teachers can support fact learning and the development of fluency by implementing aligned mastery-practice activities (Ok & Bryant, 2016; Woodward, 2006). Mastery-practice activities provide repetition and practice with quick, accurate retrieval of facts from memory. In meta-analyses of fluency interventions, drill-based mastery activities have been found to be among the most effective practices (e.g., Codding et al., 2011). Characteristics of effective mastery-practice activities include teacher modeling (Codding et al., 2011), many opportunities to respond (Kubina & Yurick, 2012), immediate affirmative or corrective feedback (Fuchs et al., 2008), and an appropriate ratio of known to unknown facts (Burns, 2005).

Activities that incorporate these elements of effective mastery practice include cover, copy, compare (CCC; e.g., Riccomini et al.; 2017; Skinner et al., 1989); incremental rehearsal (Burns et al., 2019); constant time delay (CTD;

Koscinski & Gast, 1993); taped problems (McCallum et al., 2004); and some computer-based practice programs and applications (Ok & Bryant, 2016). Each of these mastery activities is supported by research and each can be implemented in under 15 minutes once students have learned activity rules, routines, and procedures. In the following sections, we briefly summarize support for each practice and describe how teachers can implement each activity. Additional resources for implementation of mastery activities are provided in ***Table 2***.

CCC

CCC is a mastery-practice activity that students complete independently by working through a specially designed worksheet including one or two unknown facts and eight or nine known facts (Riccomini et al., 2017). CCC is considered an evidence-based practice and been used successfully to improve fluency outcomes for students with disabilities in a number of studies (e.g., Becker et al., 2009; Skinner et al., 1989). A CCC worksheet for fact practice has a column with up to 10 facts and answers on the left and two blank columns to the right (see an example by following the link to a CCC worksheet template in ***Table 2***). To begin, students read and recite the first fact and answer in the leftmost column, then fold the paper back to hide the fact and answer (i.e., cover) before writing the fact and answer in the next column (i.e., copy). Next, they unfold the paper to check their work (i.e., compare). If they made an error, they self-correct by repeating the process and writing the fact and answer again. They repeat the process until they have completed the CCC procedure for each fact on the worksheet. In order to use CCC effectively and efficiently, teachers must provide students with explicit instruction and guided practice with feedback in using the CCC procedure (Riccomini et al., 2017). For additional information on implementing CCC effectively, see ***Table 2***.

Incremental Rehearsal

Incremental rehearsal is an effective flashcard practice technique (Kupzyk et al., 2011) that provides mixed practice of previously learned and new facts at a ratio of one new or unknown fact to seven, eight, or nine unknown facts (1:7, 1:8, or 1:9; Burns et al., 2019). A practice cycle using a ratio of one unknown to eight known facts includes eight sets of facts. In each set, the new fact is presented first and is followed by an increasing number of known facts (for a visual representation of the flashcard sequence, follow the link for the Evidence Based Intervention Network's incremental-rehearsal practice guide in ***Table 2***). After students complete the cycle, the unknown fact becomes the first known fact and the final (eighth) known fact is removed from the set. Up to five new facts can be introduced during an incremental-rehearsal session. With the introduction of each new fact, students complete another cycle. Each cycle takes 3 minutes or less, so if five new facts are introduced, the entire practice session should take no longer than 15 minutes total.

When starting an incremental-rehearsal practice session, the teacher or peer tutor introduces the first unknown fact by reading it aloud and stating the answer. The student then repeats the new fact and answer twice. After the new fact has been introduced using this procedure, the teacher or tutor initiates the incremental-rehearsal practice cycle as described in the previous paragraph. If the teacher or tutor is introducing five new facts, they will work through five cycles, adding a new fact and removing the final known fact each time. If a student makes an error or does not respond to a fact within 2 to 3 seconds, the teacher or tutor provides an immediate error correction by reading the fact, stating the correct answer, then asking the student to repeat the fact and correct answer. For additional resources for implementing incremental rehearsal effectively, see ***Table 2***.

CTD

CTD combines a teaching sequence and a practice sequence using flashcards (Cooper et al., 2007). When using CTD, the teacher or tutor presents a fact on a flashcard, and if the student is unable to respond correctly in a predetermined amount of time (e.g., 2–3 seconds), the teacher provides the correct answer. During the teaching sequence, the teacher or tutor presents each flashcard in a set to the student and reads the fact and answer aloud, then immediately prompts the student to repeat the fact and answer (0-second delay). During the practice sequence, the teacher or tutor presents the fact flashcards again. Rather than provide the answer, the teacher or tutor asks the student to state the fact and answer independently using a longer delay (e.g., 2–3 seconds). If the student responds correctly within the 2 to 3 seconds provided, the teacher or tutor provides affirmative feedback (e.g., "Correct!"), then advances to the next flashcard. If the student does not respond within 2 to 3 seconds or responds incorrectly, the teacher or tutor provides corrective feedback and asks the student to repeat the correct answer (e.g., "$5 \times 6 = 30$. What is 5×6?"). For additional resources implementing CTD effectively, see ***Table 2***.

Taped Problems

Taped problems are a mastery-practice activity that can use constant or progressive time delay to provide math fact practice using practice sheets that are aligned with audio recordings of the same facts and answers (McCallum et al., 2004). During taped-problem practice, students attempt to answer facts on their worksheet before they hear the fact and answer read aloud on the tape recording (i.e., they attempt to "beat the tape"). When combining taped problems and CTD procedures, the delay between each fact prompt and answer on the audio recording is held constant across problems. When using progressive time delay, the delay between fact prompts and answers gets incrementally shorter or longer (Cooper et al., 2007). Beginning with a longer delay (4–5 seconds) and incrementally decreasing the time allotted for students to respond across trials allows students more time to retrieve fact answers from memory initially and supports the development of faster, more automatic responding with repeated practice (McCallum et al., 2004). For additional resources implementing taped problems effectively, see ***Table 2***.

Computer-Based Mastery Practice

Reviews of the literature on computer-based fact fluency programs and

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applications indicate that these tools can increase fact fluency for students with mathematics difficulties when they promote engagement and include important elements of effective practice, including ample opportunities to respond, immediate feedback and error correction features, customization features (i.e., the ability to individualize practice items or sets), and progress-monitoring features (Hawkins et al., 2017). In contrast, computer games and programs that do not contain these elements may not support growth in fact fluency. Although an up-to-date review evaluating popular math fact apps and programs for the inclusion of these features is needed, Musti-Rao et al. (2015) found that Math Drills by Instant Interactive was the only app out of three reviewed that included all recommended features. Research on the effectiveness of computer-based programs and applications for building fluency have generated mixed results, likely due in part to variability in the inclusion of important elements of effective practice features within different programs and applications. In effect, teachers should carefully consider the guidelines and recommendations provided by researchers (Hawkins et al., 2017) in selecting an app or program to provide fact practice and, after putting a program into use, should monitor student progress closely to gauge effectiveness.

One example of successful computer application use for building fact fluency comes from a study conducted by Ok and Bryant (2016). In this study, researchers effectively combined explicit strategy instruction with mastery practice provided through an iPad application called Math Evolve (Zephyr Games, 2012, cited in Ok & Bryant, 2016). Researchers selected Math Evolve because the program allowed for customization of practice sets, provided immediate affirmative or corrective feedback, required students to self-correct before moving on when they made an error, provided both cumulative and distributed practice opportunities, and allowed for customization of the time allotted for responses (Ok & Bryant, 2016). Participants in the study engaged in 5 minutes of independent fact practice using Math Evolve at the end of each intervention lesson.

Combining Strategy Instruction and Mastery Practice

Combining fact strategy instruction with mastery-practice activities is supported by research and can aid the development of both number sense and quick, accurate fact retrieval (Ok & Bryant, 2016; Woodward, 2006). Studies that have combined strategy instruction and mastery practice to improve students' fact fluency provide guidance on how to successfully combine the two approaches (Ok & Bryant, 2016; Woodward, 2006) using an explicit instruction approach (i.e., they provided clear, step-by-step teacher models with think-aloud followed by guided and independent opportunities to practice using the strategy) and visual representations.

Combining strategy instruction and mastery practice requires explicit strategy instruction followed by brief, repeated mastery-practice sessions with carefully designed practice sets including appropriate ratios of new to previously learned facts. Strategy instruction is used to build conceptual understanding, and mastery practice follows to increase students' retrieval speed and accuracy and to provide distributed practice. As an example, a teacher might choose to provide an explicit strategy lesson on Monday (e.g., the "doubles strategy" minilesson; see ***Figure 1***), then quickly review the strategy and implement a variety of brief, targeted mastery-practice activities on subsequent days of the week. Teachers might opt to use a variety of mastery activities to maintain student interest and motivation during practice. For example, they might follow the strategy lesson on Monday with a CCC practice worksheet on Tuesday, an incremental-rehearsal session on Wednesday, and a taped-problems session on Thursday. Teachers can increase the effectiveness of practice by taking the time to explicitly teach and provide practice with practice-activity rules, routines, materials, and procedures. If finding time for fluency activities during the traditional mathematics instructional period is a challenge, teachers may try implementing activities during transition times. Using centers or stations may also improve the efficiency with which teachers can implement fluency activities.

After several weeks of consistent fluency-building work focused on the "times 2" and "times 5" facts, students have demonstrated significant growth. Ms. Kent decides to reteach fraction addition with unlike denominators to her struggling students using fractions with factors of 2 and factors of 5 in the denominators. Using these carefully selected examples, students are able to apply their newly acquired fact knowledge to the complex procedure. This greatly reduces their frustration and helps improve students' procedural fluency. Seeing this success, Ms. Kent and Mr. Stephens agree to continue dedicating time throughout the week for strategy instruction and mastery practice.

Conclusion

Fact fluency is foundational for success in mathematics (NMAP, 2008). With the resources provided in this article, teachers can help students develop fact fluency by combining explicit strategy instruction and mastery-practice activities (Gersten et al., 2009; Woodward, 2006). Prioritizing effective fact fluency instruction and practice will strengthen students' foundational mathematics knowledge and pave the way for success for with more complex concepts and procedures.

DECLARATION OF CONFLICTING INTERESTS

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

FUNDING

The author(s) received no financial support for the research, authorship, and/or publication of this article.

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TEACHING Exceptional Children,
Vol. 53, No. 1, pp. 60–69.

Erratum

Withdrawal – Administrative Duplicate Publication: Supporting students with disabilities during school crises: A teacher's guide

Clarke, L. S., Embury, D. C., Jones, R., Yssel, N. (2014). Supporting students with disabilities during school crises: A teacher's guide. *Teaching Exceptional Children.* Advance online publication. doi: 10.1177/0014402914534616

Ahead of Print article withdrawn by publisher.

Due to an administrative error, this article was accidentally published Online First and in Volume 46 Issue 6 with different DOIs.

The correct and citable version of the article remains:

Clarke, L. S., Embury, D. C., Jones, R., Yssel, N. (2014). Supporting students with disabilities during school crises: A teacher's guide. *Teaching Exceptional Children, 46*(6), 169-178. https://doi.org/10.1177/0040059914534616